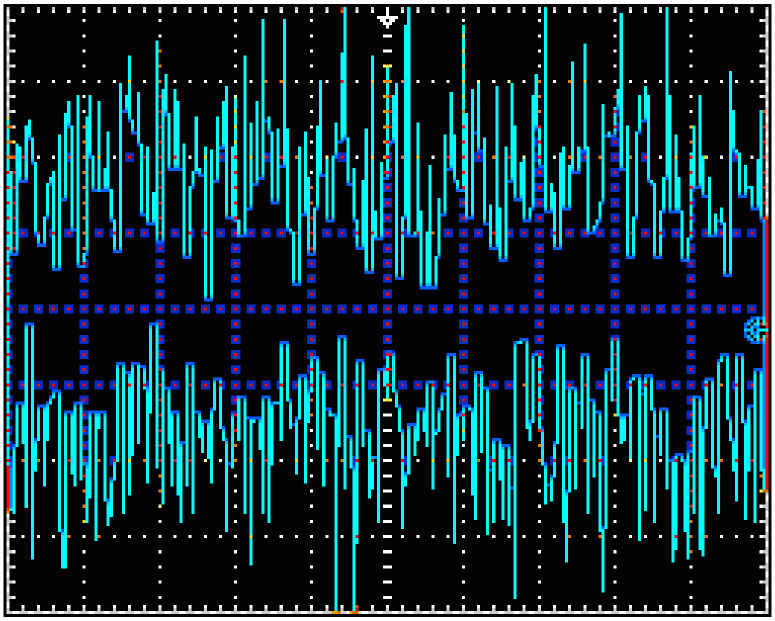
Noise Tutorial
Part III ~ Attenuator and Amplifier Noise



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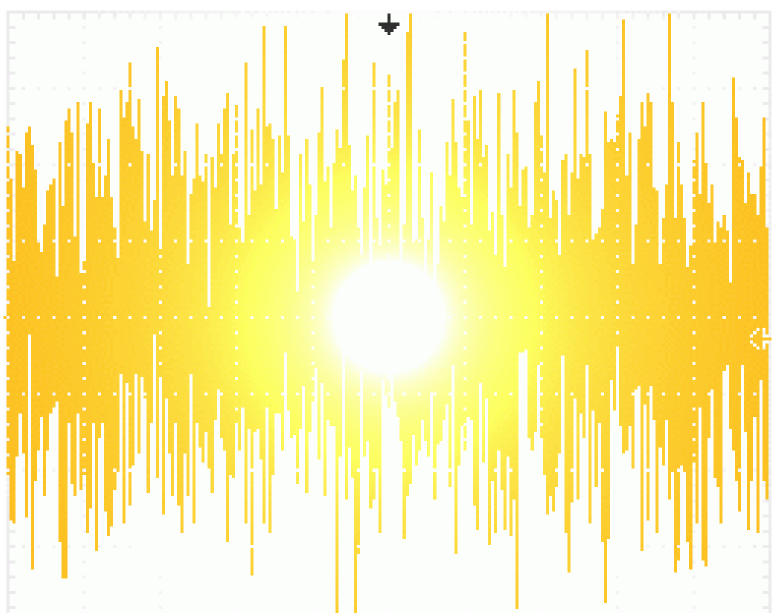

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Noise Tutorial III ~ Attenuator and Amplifier Noise

Abstract: With the exception of some solar radio bursts, the extraterrestrial emissions received on Earth's surface are very weak. Noise places a limit on the minimum detection capabilities of a radio telescope and may mask or corrupt these weak emissions. An understanding of noise and its measurement will help observers minimize its effects. This paper is a tutorial and includes six parts.

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3-1. Attenuation effects on noise temperature

All practical transmission lines (coaxial cables, waveguides, open wire) and their associated connectors introduce loss between their input and output. Poor cable installation practices and improper use and installation of connectors introduce additional losses. Problems such as these can be especially apparent at VHF and above. A transmission line or attenuator that is matched at both its input and output can be characterized as a 2-port network (figure 3-1).

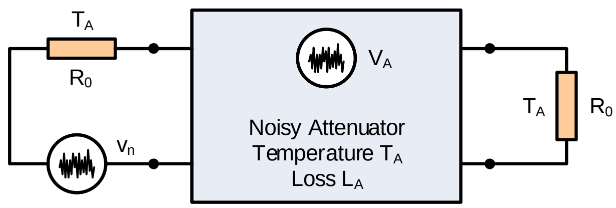


Fig. 3-1 ~ 2-port circuit used to represent an attenuator. The input is from an external noise voltage source v_n and both input and output terminations are matched. The noise voltage produced by the attenuator is represented by V_A. All components are at the same physical temperature T_A

The attenuator has loss L_A, which is defined as the linear ratio of the attenuator input power to its output power ($L_A \geq 1$). It should be noted this definition is equivalent to a positive logarithmic power ratio in dB. For example, a linear power ratio of 3.0 is equivalent to +4.81 dB. Loss in dB usually is spoken as a positive value (for example, "The cable loss is 4.81 dB."). One must be careful to use the correct sign in calculations. It also is necessary to be careful to not confuse loss with amplifier gain in dB, which also is spoken as a positive value. When cable or attenuator loss is given as a positive dB value, the loss as a linear power ratio of input to output is

$$L_A = 10^{\left(\frac{L_{A,dB}}{10}\right)} \qquad (3\text{-}1)$$

where $L_{A,dB}$ is the loss in dB.

For this analysis, all components are at physical temperature T_A. We will determine the effective noise temperature T_{eff} of the attenuator on the basis of thermal equilibrium and will take into account the equivalent noise power of the voltage source as well as the noise power contributed by the attenuator. The equivalent noise power from the source at the attenuator input is

$$N_s = k \cdot T_A \cdot B_n \qquad (3\text{-}2)$$

The noise power flowing from the attenuator to the load consists of two components, the noise power of the source N_s reduced by the attenuator and the noise power N_A contributed by the attenuator itself (figure 3-2). Therefore,

$$N_{out} = \frac{1}{L_A} \cdot N_s + N_A \qquad (3\text{-}3)$$

Note that, because of the definition of the attenuator loss L_A as the ratio of input to output power, we must invert it to obtain the fraction of N_s that appears at the attenuator output.

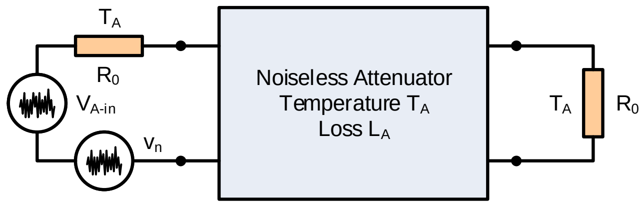


Fig. 3-2 ~ Attenuator with its internal noise voltage source V_A referred to its input and redesignated $V_{A\text{-}in}$. The noise voltage v_n of the source is connected to the attenuator input.

For thermal equilibrium, the noise power flowing back to the attenuator from the load must equal N_{out}. The total noise power of the load is

$$N_L = k \cdot T_A \cdot B_n \qquad (3\text{-}4)$$

Combining Eq. (3-3) and (3-4),

$$k \cdot T_A \cdot B_n = \frac{1}{L_A} \cdot N_s + N_A \qquad (3\text{-}5)$$

Substituting N_s from Eq. (3-2)

$$k \cdot T_A \cdot B_n = \frac{1}{L_A} \cdot \left(k \cdot T_A \cdot B_n\right) + N_A \qquad (3\text{-}6)$$

Solving for N_A

$$N_A = k \cdot T_A \cdot B_n - \frac{1}{L_A} \cdot k \cdot T_A \cdot B_n = \left(\frac{L_A - 1}{L_A}\right) \cdot k \cdot T_A \cdot B_n \qquad (3\text{-}7)$$

Let T_{eff} represent the effective noise power of the attenuator referred to its input. This is reduced by the attenuator, so the attenuator noise power at its output is

$$N_A = \frac{1}{L_A} \cdot k \cdot T_{eff} \cdot B_n \qquad (3\text{-}8)$$

Combining Eq. (3-7) and (3-8) gives

$$\frac{1}{L_A}\cdot k\cdot T_{eff}\cdot B_n=\left(\frac{L_A-1}{L_A}\right)\cdot k\cdot T_A\cdot B_n \tag{3-9}$$

Cancelling terms and rearranging,

$$T_{eff}=T_A\cdot\left(L_A-1\right) \tag{3-10}$$

Example 3-1:
Determine the effective noise temperature of an attenuator with losses a) 1.0, b) 1.25 and c) 10.0 (all linear power ratios). The attenuator physical temperature is 300 K.

Solution:
a) $T_{eff}=T_A\cdot\left(L_A-1\right)=300\cdot\left(1.0-1\right)=0\ \text{K}$

b) $T_{eff}=300\cdot\left(1.25-1\right)=75\ \text{K}$

c) $T_{eff}=300\cdot\left(10-1\right)=2700\ \text{K}$

Comment: The transmission line in a) is lossless and, as expected, its effective noise temperature is 0 K; that is, it adds no noise to the system. In b) the loss is equivalent to about 1 dB and the noise power added by the transmission line has a temperature of 75 K. In c) the loss is equivalent to 10 dB and the added noise power has a temperature of 2700 K, a factor of 36 times higher than b). With large attenuation values, the effective temperature can be quite high. However, the resulting noise power is reduced by the attenuator and the noise temperature at its output never rises above the attenuator physical temperature. However, it is the effective noise temperature that determines the attenuator noise factor as described later.

All devices, including filters, attenuators, amplifiers, and transmission lines, have an effective noise temperature and can be evaluated as previously described. For example, consider an antenna that receives emissions with total noise temperature T_{Ant} and is connected to a transmission line with physical temperature T_L and loss L_L. Using the concepts discussed above for an attenuator, the transmission line effective noise temperature T_{eff}

$$T_{eff}=T_L\cdot\left(L_L-1\right) \tag{3-11}$$

The total noise power from the antenna and transmission line at the output of the transmission line is

$$N_{out}=\frac{1}{L_L}\cdot\left(k\cdot T_{Ant}\cdot B_n+k\cdot T_{eff}\cdot B_n\right) \tag{3-12}$$

Substituting the effective noise temperature T_{eff} from Eq. (3-11) and simplifying

$$N_{out} = \frac{1}{L_L} \cdot \left(k \cdot T_{Ant} \cdot B_n + \left(L_L - 1 \right) \cdot k \cdot T_L \cdot B_n \right) \tag{3-13}$$

In terms of total noise temperature T_{Total} the total noise power is

$$k \cdot T_{Total} \cdot B_n = \frac{1}{L_L} \cdot k \cdot T_{Ant} \cdot B_n + \left(\frac{L_L - 1}{L_L} \right) \cdot k \cdot T_L \cdot B_n \tag{3-14}$$

By cancelling terms, the total effective noise temperature of the antenna including the transmission line contribution is

$$T_{Total} = \frac{1}{L_L} \cdot T_{Ant} + \left(\frac{L_L - 1}{L_L} \right) \cdot T_L \tag{3-15}$$

The noise temperature of the received emissions is reduced by the factor $\frac{1}{L_L}$ and the transmission line itself increases the noise temperature as much as T_L depending on its loss.

3-2. Amplifier noise

Noise is generated within the active and passive components and power supplies of any practical amplifier. Therefore, the amplifier itself adds noise to the noise power applied to the amplifier input from an external source. The input noise is amplified and the noise at the output of each stage of a multi-stage amplifier is amplified again by the stages following it. Any noise in the early stages of a multi-stage amplifier experiences considerably more amplification than later stages. This leads to the interesting conclusion, discussed in more detail later, that the early stages of an amplifying system contribute the most to a receiver's noise performance.

The noise performance of an amplifier can be analyzed by connecting its input to an external noise source with noise power N_S (figure 3-3). The source noise power can be represented by a noise temperature T_S. T_S is not necessarily a physical temperature but accounts for the noise power available from the source. The source can be an antenna, signal generator, noise generator or a previous amplifier stage. The amplifier itself contributes its internal noise power N_{amp} to its output. The external noise source is amplified according to the amplifier gain G; therefore, the total output noise power is

$$N_{out} = G \cdot N_S + N_{Amp} \tag{3-16}$$

It is convenient to refer the amplifier noise to its input. For this situation, both the noise source N_S and amplifier noise now designated $N_{Amp\text{-}in}$ are amplified, and the total output noise power is

$$N_{out} = G \cdot \left(N_S + N_{Amp-in} \right) \tag{3-17}$$

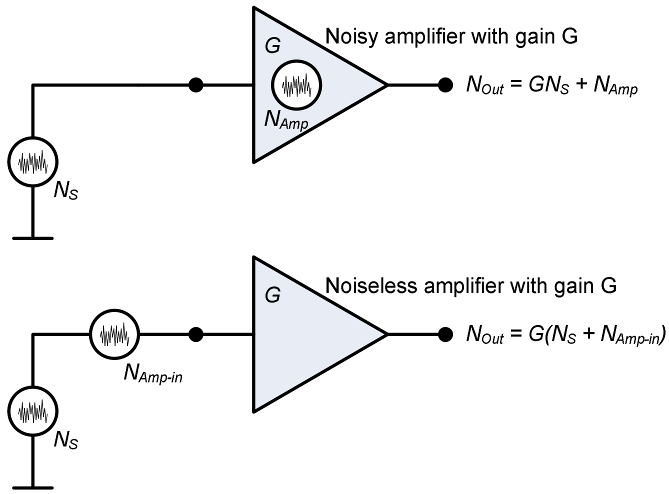


Fig. 3-3 ~ Amplifier input noise. Upper: An external noise source is connected to the amplifier input, where it is amplified and combined with the noise generated within the amplifier. Lower: The internal amplifier noise can be referred to the amplifier input, where it is combined with the external noise and both are then amplified by a noiseless amplifier.

Assume the amplifier has an input resistance R_{Amp} that is matched to the noise source resistance R_S. The many individual noise sources inside the amplifier are represented by an equivalent noise temperature T_{Amp}, which is a measure of the noise added to the input by the amplifier. The amplifier has power gain G. The output of the amplifier is connected to an ideal (noiseless) load resistance R_L (figure 3-4).

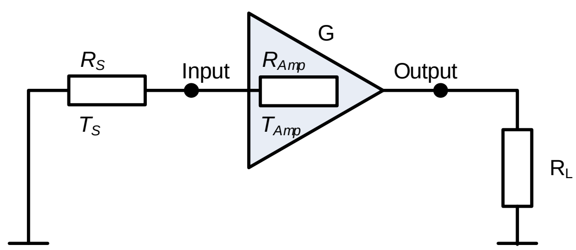


Fig. 3-4 ~ Amplifier with gain G connected to a noiseless load resistor R_L

First, we consider only the noise available from the two sources, R_S and R_{Amp}. We can replace the two resistors with their noise models (figure 3-5). To determine the noise contribution from these resistors, we have to make two calculations, one associated with R_S and the other associated with R_{Amp}. We determine the noise power contributed by the first resistor by shorting out the noise voltage source associated with the second resistor, and then repeat the process for the second resistor. The analysis is simplified by the fact that R_S and R_{Amp} are equal (matched), ensuring that the noise voltage divides equally across the two resistors.

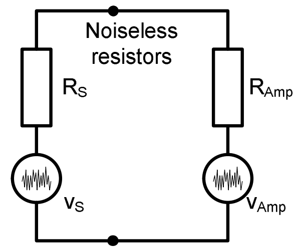


Fig. 3-5 ~ Noise model of the amplifier input circuit

The noise power available from R_S with v_{Amp} shorted is

$$N_S = k \cdot T_S \cdot B_n \tag{3-18}$$

This power is dissipated in both R_S and R_{Amp} because both have the same voltage and same resistance. The noise power available from R_{Amp} with v_S shorted is

$$N_{Amp-in} = k \cdot T_{Amp-in} \cdot B_n \tag{3-19}$$

Again, this power is dissipated in both R_S and R_{Amp}. The total power dissipated by R_{Amp} and R_S is

$$N_{Total-in} = N_S + N_{Amp-in} = \left(k \cdot T_S \cdot B_n\right) + \left(k \cdot T_{Amp-in} \cdot B_n\right) \tag{3-20}$$

Rearranging terms

$$N_{Total-in} = k \cdot B_n \cdot \left(T_S + T_{Amp-in}\right) \tag{3-21}$$

The total effective temperature at the amplifier input is found from

$$k \cdot B_n \cdot T_{Total} = k \cdot B_n \cdot \left(T_S + T_{Amp-in}\right) \tag{3-22}$$

Cancelling terms

$$T_{Total} = T_S + T_{Amp-in} \tag{3-23}$$

Eq. (3-23) provides an important result: In a matched system the total amplifier noise temperature depends only on the sum of the source and amplifier noise temperatures. At the amplifier output, the noise power is increased by the amplifier gain, G, or

$$N_{Total-Out} = G \cdot k \cdot B_n \cdot \left(T_S + T_{Amp-in}\right) \tag{3-24}$$

and it follows that

$$T_{Total-Out} = G \cdot \left(T_S + T_{Amp-in}\right) \quad (3\text{-}25)$$

The amplifier gain shifts the reference point for the input and amplifier noise to the amplifier output. We can plot the relationship between the output noise power and the input noise temperature to show how the amplifier adds noise to the system (figure 3-6). As the input noise temperature is reduced to zero, the output noise decreases but does not reach zero. The output noise power at the vertical axis intercept represents the noise contribution of the amplifier, N_{Amp}. A low noise amplifier will intercept the vertical axis at a lower point than an ordinary or noisier amplifier.

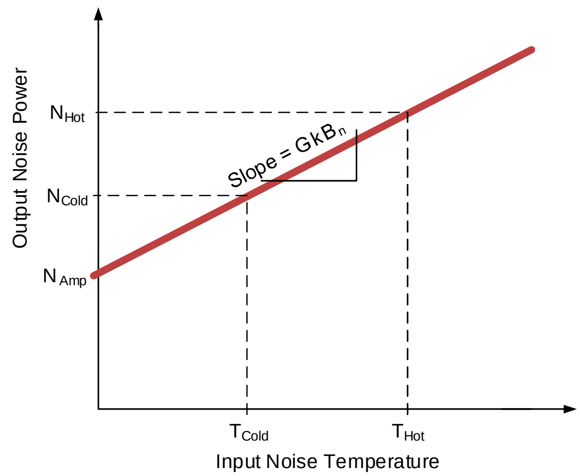


Fig. 3-6 ~ Output noise power as a function of input noise temperature. Two arbitrary input temperature points are shown, T_{Cold} and T_{Hot} with corresponding output noise powers N_{old} and N_{Hot}. The straight plot line has a slope $G \cdot k \cdot B_n$ and intersects the vertical axis where the input noise temperature T_S = 0 K. Even when there is no input noise from an external source, the amplifier adds noise and the output noise power is N_{Amp}.

3-3. Cascaded amplifiers

Each amplifier in a cascade adds noise (figure 3-7). It was found previously that the total noise power at the output of an amplifier is determined by the noise power at its input plus the amplifier noise referenced to its input, both increased by the amplifier gain. Therefore, at the output of the first amplifier and input of the second amplifier,

$$N_{Out-1} = G_1 \cdot k \cdot B_n \cdot \left(T_1 + T_S\right) \quad (3\text{-}26)$$

where

T_1 noise temperature of amplifier 1 referenced to input

G_1 gain of amplifier 1

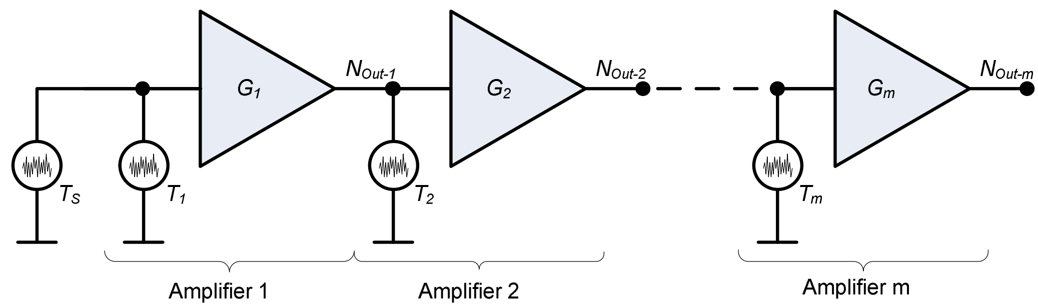


Fig. 3-7 ~ Cascade of m amplifiers, each with respective noise temperatures T_1, T_2, . . . T_m and gains G_1, G_2, . . . G_m

Assuming matched conditions, the output of the second amplifier consists of the noise power at its input from the first amplifier plus the noise power added by the second amplifier, or

$$N_{Out-2} = G_2 \cdot \left[k \cdot B_n \cdot T_2 + G_1 \cdot k \cdot B_n \cdot \left(T_1 + T_S \right) \right] \tag{3-27}$$

Rearranging

$$N_{Out-2} = k \cdot B_n \cdot G_2 \cdot \left[T_2 + G_1 \cdot \left(T_1 + T_S \right) \right]$$
$$N_{Out-2} = k \cdot B_n \cdot \left[G_2 \cdot T_2 + G_1 \cdot G_2 \cdot \left(T_1 + T_S \right) \right] \tag{3-28}$$

This process can be carried through as many stages of amplification as necessary; for the mth stage

$$N_{Out-m} = k \cdot B_n \cdot \left\{ \left(G_m \cdot T_m \right) + \ldots + \left(G_3 \cdots G_m \cdot T_3 \right) + \left(G_2 \cdot G_3 \cdots G_m \cdot T_2 \right) + \left[G_1 \cdot G_2 \cdot G_3 \cdots G_m \cdot \left(T_1 + T_S \right) \right] \right\} \tag{3-29}$$

We can obtain the cascade's equivalent input noise power by dividing the output power by the total cascade gain, or

$$N_{in} = \frac{N_{Out-m}}{G_1 \cdot G_2 \cdot G_3 \cdots G_m}$$
$$N_{in} = k \cdot B_n \cdot \frac{\left(G_m \cdot T_m \right) + \ldots + \left(G_3 \cdots G_m \cdot T_3 \right) + \left(G_2 \cdot G_3 \cdots G_m \cdot T_2 \right) + \left[G_1 \cdot G_2 \cdot G_3 \cdots G_m \cdot \left(T_1 + T_S \right) \right]}{G_1 \cdot G_2 \cdot G_3 \cdots G_m} \tag{3-30}$$

Simplifying and rearranging

$$N_{in} = k \cdot B_n \cdot \left(T_S + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} + \cdots + \frac{T_m}{G_1 \cdot G_2 \cdot G_3 \cdots G_{m-1}} \right) \tag{3-31}$$

If we let T_{equiv} be the equivalent input noise temperature of the cascaded amplifiers as a whole and note that $N_{in} = k \cdot B_n \cdot T_{equiv}$, then

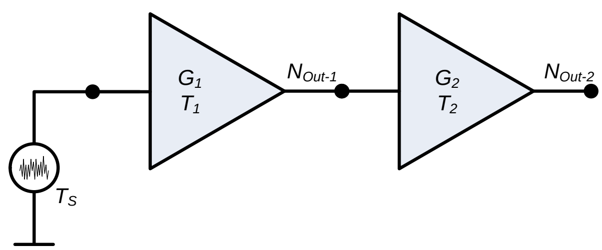
$$T_{equiv} = T_S + T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} + \cdots + \frac{T_m}{G_1 \cdot G_2 \cdot G_3 \cdots G_{m-1}} \tag{3-32}$$

If we are concerned only with the total equivalent noise power of the cascaded amplifiers, not including the input noise power due to the source, T_S can be dropped from the equation, giving

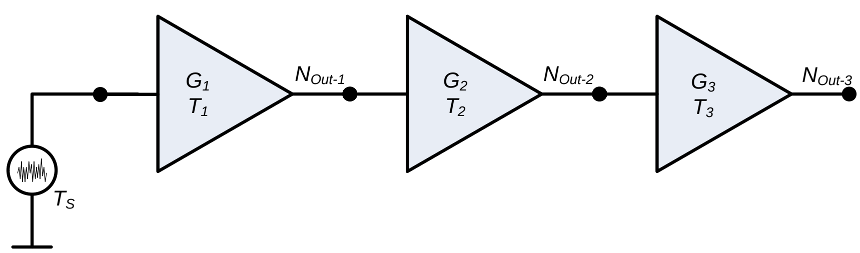
$$T_{Cascade} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} + \cdots + \frac{T_m}{G_1 \cdot G_2 \cdot G_3 \cdots G_{m-1}} \tag{3-33}$$

Examination of this expression leads to an interesting conclusion if the individual amplifier gains are high: The first stage noise temperature (T_1) can be an important controlling factor in receiver system noise performance and subsequent stages have considerably less importance. The second stage noise temperature is reduced by the factor $1/G_1$ and the third stage by the factor $1/(G_1 \cdot G_2)$. For example, if the power gain of each stage is 50 (17 dB), then the noise temperature of the second stage is reduced by the factor 0.02 and the third stage by the factor 0.0004.

Example 3-3:
Find the equivalent noise temperature of cascaded amplifiers shown (figure 3-8) for the conditions given.



(a) Two amplifiers in cascade: G_1 = 60 dB, G_2 = 20 dB, T_1 = 300 K, and T_2 = 20,000 K



(b) Three amplifiers in cascade: G_1 = 30 dB, G_2 = 40 dB, G_3 = 50 dB and T_1 = 50,000 K, T_2 = 20,000 K and T_3 = 30,000 K

Fig. 3-8 ~ Equivalent noise temperature of cascaded amplifiers

Solutions:

(a) First, convert the gains to linear ratios,

$$G_1 = 10^{\frac{G_1(dB)}{10}} = 10^{\frac{60}{10}} = 10^6 \text{ and } G_2 = 10^{\frac{G_2(dB)}{10}} = 10^{\frac{20}{10}} = 10^2$$

Using Eq. (2-59)

$$T_{Cascade} = T_1 + \frac{T_2}{G_1} = 300 + \frac{20,000}{10^6} = 300.02 \text{ K}$$

(b) Converting as above,

$$G_1 = 10^{\frac{30}{10}} = 10^3 \text{ , } G_2 = 10^{\frac{40}{10}} = 10^4 \text{ and } G_3 = 10^{\frac{50}{10}} = 10^5$$

and

$$T_{Cascade} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} = 50,000 + \frac{20,000}{10^3} + \frac{30,000}{10^3 \cdot 10^4} = 50,020.003 \text{ K}$$

Comment: In both examples, the equivalent temperature is largely determined by the noise temperature of the first stage indicating that the noise performances of subsequent stages have little influence on overall amplifier noise performance.

3-4. References

There are no references in this part.

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