REEVE ENGINEERS

Sound Advice In Telecommunications

The DeciBel

1. Introduction

This application note covers the logarithmic unit of signal ratio most commonly used in telephony, the deciBel (dB). Another related application note covers the application of the dB in using Transmission Level Points.[1]

A sound increase from one power level to another heard by the human ear leaves an impression of increased loudness that is approximately proportional to the logarithm of the ratio of the two powers. The original unit given to this phenomenon was the "Bel." However, it was not convenient to use and is not seen in practical problems. This gave rise to the "deciBel," which is Bel/10.

A change in power level of 1 dB is barely perceptible to the human ear, while a change of 2 dB is noticeable. A change of 3 dB represents one-half our twice the power.

2. Definition of dB

By definition, the deciBel, or dB, is the logarithmic (base 10) ratio of two powers, P_1 and P_2 given by

$$dB = 10\log\left(\frac{P_2}{P_1}\right)$$

> Example:
> Given that $P_2 = 1$ mW and $P_1 = 0.4$ mW, then $P_2/P_1 = 2.5$, and dB $= 10\log(2.5) = 3.98$ dB

The sign resulting from the calculation indicates which power is greater. If the sign is positive, then $P_2 > P_1$; if negative, then $P_2 < P_1$. It is important that the two powers be expressed in the same units, such as milliWatt (mW) or Watt (W), otherwise the calculation is meaningless.

If the number of dB is known, the power ratio can be found from

$$\left(\frac{P_2}{P_1}\right) = Anti\log\left(\frac{dB}{10}\right) = 10^{\frac{dB}{10}}$$

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[1] See Application Note No. 8, *Transmission Level Point.*

Example:
Given that dB = -3.98 dB, then $P_2/P_1 = 10^{-0.398} = 0.4 = 1/2.5$

A change in level of 1 dB is barely perceptible to the ear, and a change of 2 dB is only slightly apparent.

3. Variations of the dB

Some common variations of the dB used in telecommunications are:

dBm   dB relative to one milliwatt ($P_1 = 1$ mW)
dBm0   dBm referred to the 0 Transmission Level Point (0 TLP)
dBrn   dB relative to one picoWatt reference noise level ($P_1 = 1$ pW Reference Noise)
dBrnC   dBrn measured with a C-message filter or with C-notched frequency weighting filter
dBrnC0   dBrnC referred to the 0 TLP
dBdsx   dB relative to the level at a digital signal cross-connect (DSX) point (P_1 = power expected at the Digital Signal Cross-Connect)

In the above list, power is measured relative to some standard value, such as 1 mW or 1 pW. Therefore, -90 dBm is a power 90 dB below 1 mW, and +90 dBm is 90 dB above 1 mW. The power 0 dBm is equal to 1 mW. The noise reference power of 1 pW is 90 dB below 1 mW, or 1 pW = -90 dBm. Power measurements of DS1 rate digital circuits in dBdsx are based on ±3.0 volt pulses of a specific shape at a digital signal cross-connect jack set meeting industry standards.

Other reference powers are used in communications besides the mW or pW. For example, 1 W is frequently used in radio communications, giving measurements in dBW.

4. Applications of the dB

The practical use of dB comes from its logarithmic nature, which permits the large power ranges used in communications work to be expressed and manipulated in convenient numbers. See Table 1.

With dB, all arithmetic operations are either addition or subtraction. Signal-to-noise ratio (SNR) is an example of a simple dB calculation. SNR is the signal S(dBr) minus the noise N(dBr), where the r in dBr means some reference such as 1 mW. If the reference is 1 mW, then SNR(dB) = S(dBm) - N(dBm).

Example:
Let S = -1.6 dBm and N = -28.2 dBm

Then SNR = (-1.6) - (-28.2) = 26.6 dB

A comparable calculation when the power levels are in (say) mW, requires a division operation.

Let S = 0.69 mW and N = 0.0015 mW, the

$$SNR = 0.69 \text{ mW}/0.0015 \text{ mW} = 460$$

Converting to dB

$$dB = 10\log(460) = 26.6 \text{ dB}$$

Table 1
Power Ratios in dB

	Power Ratio	
dB	**Gain**	**Loss**
0.0	1.0	1.0
1.0	1.26	0.794
2.0	1.58	0.631
3.0	1.99	0.501
6.0	3.98	0.251
10.0	10.0	0.100
20.0	100.0	0.010
30.0	1,000.0	0.001
60.0	1,000,000.0	0.000,001

Gain and Loss is another example of how simple dB calculations can be. Using dB, gain is simply the difference between the output power and the input power of a circuit, or

$$Gain(dB) = P_{out}(dBm) - P_{in}(dBm)$$

Loss is the difference between the input power and output power of a circuit. Stated another way, loss is negative gain.

$$Loss(dB) = -Gain(dB) = P_{in}(dBm) - P_{out}(dBm)$$

Using comparable absolute power levels

$$Circuit\ Gain = P_{out}(mW)/P_{in}(mW)$$

and

$$Circuit\ Loss = 1/Gain = P_{in}(mW)/P_{out}(mW)$$

It must be remembered that readings on a Transmission Measuring Set (TMS) are in dBm and do not directly indicate gain or loss. For example, if the input power to a circuit with 2.5 dB of loss is 0.0 dBm, then the power measured on the TMS at the output is -2.5 dBm. In this example, the TMS indicated -2.5 dBm, which corresponds to a circuit loss of +2.5 dB. If the input is -13.0 dBm, the measured output power will be -15.5 dBm, which corresponds to the same circuit loss of +2.5 dB.

When subtracting two dB levels using the same reference (for example, dBm), the result is a number in dB.

> Example:
> > Measured P_{in} = 0.0 dBm and P_{out} = -9.0 dBm
> > Circuit loss = 0.0 dBm - (-9.0 dBm) = +9.0 dB

> Example:
> > Measured P_{in} = -13.0 dBm and P_{out} = -22.0 dBm
> > Circuit loss = -13.0 dBm - (-22.0 dBm) = +9.0 dB

> Example:
> > Measured P_{in} = -13.0 dBm and P_{out} = -10.0 dBm
> > Circuit loss = -13.0 dBm - (-10.0 dBm) = -3.0 dB

> Because the circuit loss in this example is negative, the circuit actually has gain [Gain(dB) = -Loss (dB)]

It is imperative that the same reference power is used in calculations involving SNR and Gain/Loss. In telecommunications, this normally means that all powers be expressed in dBm.

A typical voice frequency performance specification may require that the loss over a frequency range be between +X and -Y dB with respect to the loss at a reference frequency (usually 1004 Hz). The + means <u>more</u> loss and - means <u>less</u> loss, so that X dB more loss and Y dB less loss would be acceptable at the measurement frequency.

> Example:
> > A specification requires that the loss at a measurement frequency be within +2.5 dB and -1.0 dB of the loss measured at 1004 Hz. If the loss at 1004 Hz is found to be 9.0 dB, then the allowable loss at other frequencies is between +8.0 dB (-1.0 dB, or 1.0 dB less loss) and +11.5 dB (+2.5 dB, or 2.5 dB more loss)

> Example:
> > A measurement at 3004 Hz gives P_{out} = -22.5 dBm when P_{in} = -13.0 dBm. Find the loss at 3004 Hz and compare with the specification given in the previous example.

> > Loss = -13.0 dBm - (-22.5 dBm) = +9.5 dB

> > This loss is acceptable because the limits previously given were +8.0 to +11.5 dB

Since noise levels usually are measured relative to the noise reference power of 1 pW (0 dBrn) and not 1 mW (0 dBm), they first must be converted to dBm using the following expression:

> dBm = dBrn - 90 dB

Example:
The noise is measured as 31 dBrn. The equivalent level in dBm is

31 dBrn - 90 dB = -59 dBm

Noise measurements are made with special filters, which are described in the next section.

5. Noise Filters

Noise measurements normally are made with a filter to limit the bandwidth of the measurement. In voice frequency measurements, the C-message filter is most commonly used. In fact, this is the only filter presently specified in American National Standards Institute (ANSI) telecommunication standards for voice frequency circuit performance measurements.

Other filters that may be encountered in telecommunication measurements are listed below in order of ascending bandwidth. Some of these filters are obsolete or have been replaced by current standards, as indicated. The Institute of Electrical and Electronic Engineers (IEEE) specifies filter requirements in IEEE Std. 743-19684 and IEEE Std. 743-1995. A detailed description and application of the noise filters can be found in another application note.[2] Table 1 summarizes the filter types. When measurements are made with these filters, the noise meter readings are relative to the reference noise level of 1 pW (-90 dBm) and the nomenclature dBrnX is used, where X = filter type (for example, dBrnC for the C-Message filter).

Table 2
Noise Filters

Filter Type	Nomenclature	Remarks
C-Message	dBrnC	
D	dBrnD	New
3 kHz Flat	dBrn3kHz	Replaced by the 3.4 kHz Flat filter in IEEE Std. 743-1995
3.4 kHz Flat	dBrn3.4kHz	New
Program	dBrnProgram	Obsolete
15 kHz Flat	dBrn15kHz	
50 kilobit	dBrn50kb	Replaced by the E filter in IEEE Std. 743-1995
E	dBrnE	New
F	dBrnF	New
G	dBrnG	New

6. Voltage and Current Ratios in dB

Strictly speaking, only power ratios can be measured using dB. It is customary, though, to express the ratio of two voltages or two currents in dB also. This must be done with caution since the expression is only valid when the voltages or currents are measured across identical or conjugate

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[2] See Application Note No. 10, *Noise in Telecommunication Systems.*

impedances. If the impedances are not the same, correction factors must be used. The relationship between power and voltage is

$$P_1 = \frac{V_1^2}{Z_1} \text{ and } P_2 = \frac{V_2^2}{Z_2}$$

where

V_1 and V_2 = the voltages across impedances Z_1 and Z_2
P_1 and P_2 = the resulting powers

Taking the ratio of P_2 and P_1, as above, and assuming the impedances are resistive

$$dB = 10\log\left(\frac{P_2}{P_1}\right) \text{ or}$$

$$dB = 10\log\left[\left(\frac{V_2^2}{R_2}\right) \bigg/ \left(\frac{V_1^2}{R_1}\right)\right]$$

Rearranging terms gives

$$dB = 10\log\left[\left(\frac{V_2}{V_1}\right)^2\right] + 10\log\left(\frac{R_1}{R_2}\right) \text{ or}$$

$$dB = 20\log\left(\frac{V_2}{V_1}\right) + 10\log\left(\frac{R_1}{R_2}\right)$$

A similar expression can be developed for two currents. Assuming the impedances are the same ($R_1 = R_2$), then $R_1/R_2 = 1$. Since $\log(1) = 0$,

$$dB = 20\log\left(\frac{V_2}{V_1}\right)$$

7. Power Addition in dB

When two tones are placed on a circuit at the same time, their power levels combine. If the power levels are given in milliWatts, then simple linear addition may be used to arrive at the new power level.

Example:
If one tone has a power level of 1 mW and the other has a level of 2 mW, the total combined power is (1 mW + 2 mW =) 3 mW.

However, if the power levels are instead given in units of dB (such as dBm), then simple addition cannot be used. The following example illustrates this point.

Example:
In the previous example, the first power level was given as 1 mW. This is equivalent to 0 dBm. The second power level was 2 mW, which is equivalent to +3 dBm. Simply adding these together gives +3 dBm. Working backwards, this is equivalent to $10^{\frac{3}{10}} = 2.0$ mW. However, the correct value of 3 mW, which is equivalent to $(10\log(3) =)$ 4.77 dBm, was found previously. In this example, the correct value is higher by (4.77 dBm - 3 dBm =) 1.77 dB.

The above example shows that a somewhat different process is required to add powers when they are given in units of dB. The procedures are simple but somewhat tedious. First, the power levels in dBm are converted to mW as previously shown. Then the two powers (in mW) are added together and converted back to dBm, again as previously shown.

Example:
A power level of -10 dBm is to be added to a power level of -13 dBm. First, each level is converted to mW as follows:

$$Tone1 = 10^{\frac{-10}{10}} = 0.100 \text{ mW}$$

$$Tone2 = 10^{\frac{-13}{10}} = 0.050 \text{ mW}$$

$$Tone1 + Tone2 = 0.150 \text{ mW}$$

$$\text{Total Power of Tone1 and Tone2} = 10\log(0.150) = -8.24 \text{ dBm}$$

Note that the total power is 1.76 dB higher than the highest single power level; that is, -8.24 dB - (-10 dBm) = 1.76 dB. Within limits of rounding error, this is the same difference that we had in the previous example. Also note that the difference in power levels is 3 dB in both examples. This is no coincidence. Regardless of the absolute power levels, a given difference in dB between the two power levels will always give the same value to be added to the largest of the two. The actual amount to be added can be found from the following expression:

$$dB(adder) = 10\log\left(1 + 10^{\frac{\Delta}{10}}\right) - \Delta$$

where Δ = Difference in dB between the higher and lower power levels (always a positive number)

The dB(adder) is always added to the higher of the two power levels.

Example:
Two powers, +2 dBm and -2 dBm, are to be combined. The difference is +2 dBm - (-2 dBm) = 4 dB

$$dB(adder) = 10\log\left(1 + 10^{\frac{4}{10}}\right) - 4 = 5.46 - 4 = 1.46$$

Therefore, the total power level is +2 dBm + 1.46 dB = 3.46 dBm.

The above method is somewhat tedious and impossible without a calculator or table of logarithms and anti-logarithms. It is much simpler to solve for the combined power using the graph in Fig. 1. The graph can be used to find the adder to the larger power (on the vertical scale) when the difference between the two power levels is entered on the horizontal scale.

    <u>Example:</u>
   Tone1 = -29 dBm and Tone2 = -24 dBm
        The difference is [-24 dBm - (-29 dBm)] = 5 dB

        On the horizontal scale find 5 dB and read up until the curve is intersected. Read across to the vertical scale and find 1.2 dB. This is added to the larger of the two powers to give (-24 dBm + 1.2 dB) = -22.8 dBm for the combined power.

When more than two powers are to be combined, the procedure is to combine two at a time. In other words, any two of the powers are combined using the above procedures. The new power in dBm is added to the third power using the same procedure, and so on, until all powers are combined.

Fig. 1
Combining Two Powers in dB
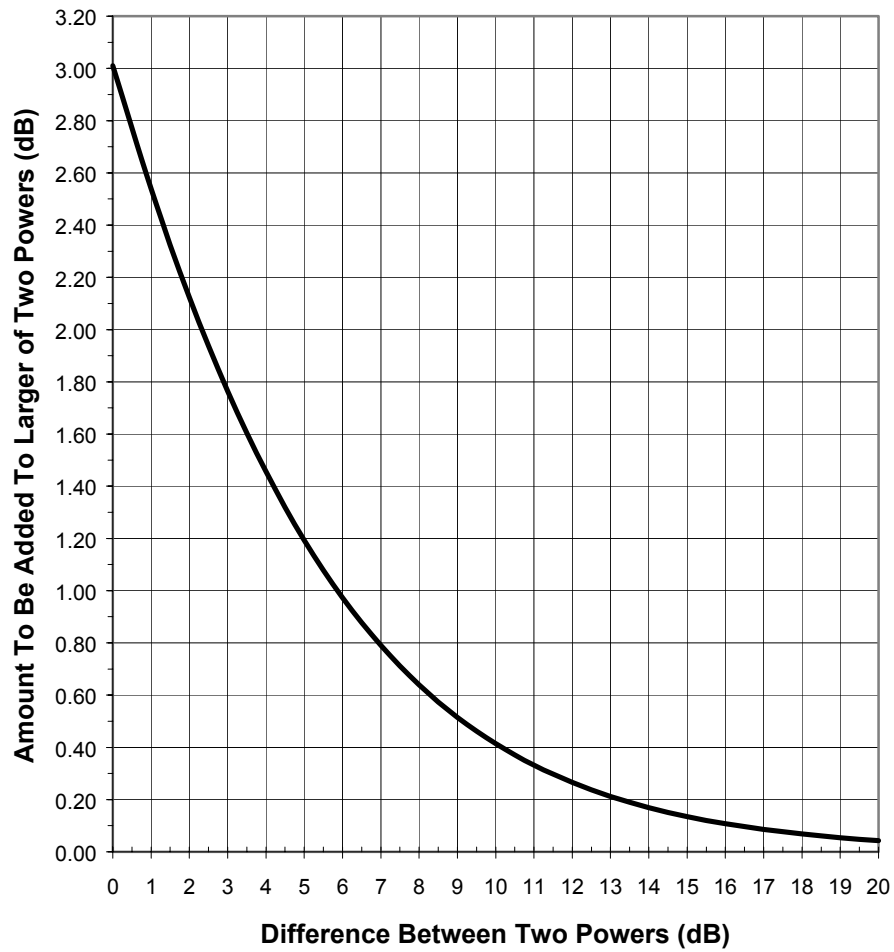
5. References

[1] Reeve, W., Subscriber Loop Signaling and Transmission Handbook: Analog, IEEE Press, 1992.

Availability: IEEE Press books are available from IEEE Press by calling 1+800-678-IEEE.

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Document Information:

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Revision: 1 (original issue, October 6, 1996)
                  2 (minor corrections, converted to PDF, November 1, 2003)

File: Decibel No7.doc