# Antenna Application for the Quadrature Coupler

Whitham D. Reeve

## 1. Introduction

The *quadrature coupler* is a 4-port device used in many radio frequency applications including mixers, amplifiers and antenna systems. It is a form of directional coupler and also is called *3 dB hybrid coupler* and *90° hybrid* or just *hybrid*. One application is to combine the outputs from two transmitters or the inputs to two receivers for connection to one antenna while simultaneously isolating the transmitters or receivers from each other. In the application described in this article, the quadrature coupler is used with the Long Wavelength Array (LWA) crossed-dipole antenna (figure 1) to receive and discriminate circularly polarized radio waves. The LWA antenna and other crossed-dipole antennas are used in the e-Callisto solar radio spectrometer network. See {e-Callisto} for additional details on e-Callisto and [ReeveLWA] for additional details on the LWA antenna.

Note: Links in braces { } and references in brackets [ ] are provided in **section 8**.


Figure 1 ~ Long Wavelength Array crossed-dipole antenna. The antenna consists of two independent tied-fork dipoles with their vertical planes perpendicular to each other. Each blade (or wing) is sloped 45°to improve response to radio waves whose plane of polarization is not aligned with the antenna axis. The horizontal components are fiberglass rod braces. The frequency range of the LWA antenna is approximately 10 to 90 MHz. (Image © 2014 W. Reeve)

## 2. Quadrature Coupler Characteristics

The quadrature coupler has a number of important characteristics: It is symmetrical; it equally splits the input signal and couples to two output ports (the signal on each output is nominally 3 dB lower than the input); it provides 0° and 90° phase shift to the output ports; and it has a port that is isolated from the input port (figure 2 and 3). Because the device is symmetrical, any port can be used as an input and the other ports provide isolation and phase shift (table 1). The analyses in the following sections are based on using ports 1 and 2 as inputs.

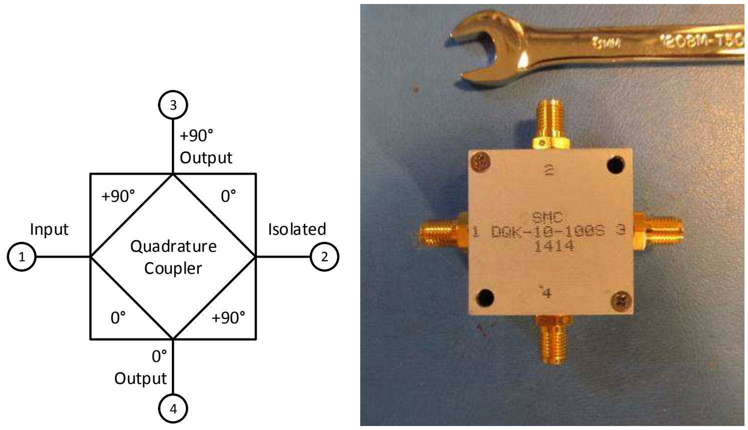


Figure 2 ~ Quadrature coupler conceptual schematic (left). The labels shown in the schematic correspond to the Synergy DQK-10-100S coupler (right) shown with an 8 mm wrench. Device dimensions are 1.25 in square x 0.75 in high and frequency range is 10 to 100 MHz. It is equipped with SMA-F connectors. (Images © 2014 W. Reeve)

Table 1 ~ Input and output combinations for the DQK-10-100S quadrature coupler

| Input | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | Input | Isolated | +90° Output | 0° Output |
| 2 | Isolated | Input | 0° Output | +90° Output |
| 3 | +90° Output | 0° Output | Input | Isolated |
| 4 | 0° Output | +90° Output | Isolated | Input |

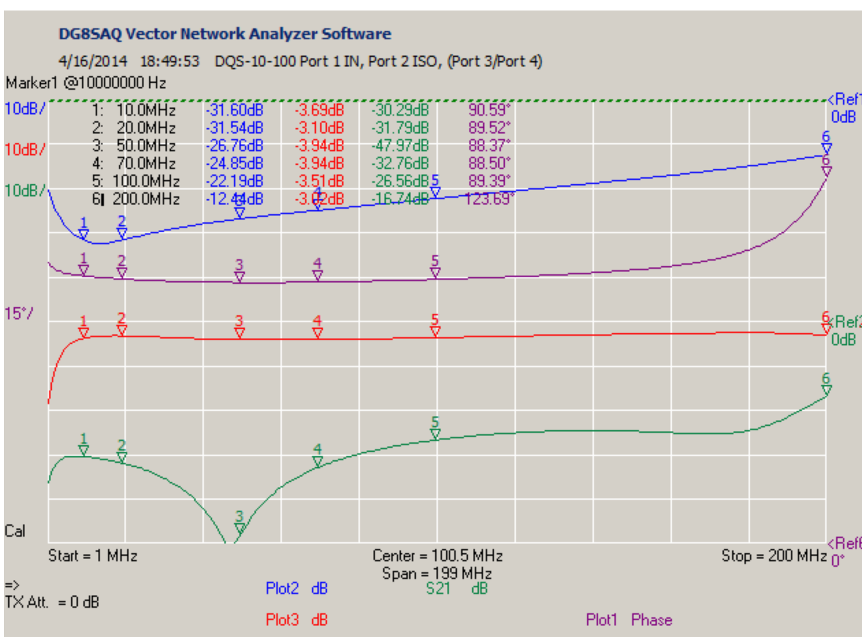


Figure 3 ~ Measurements from 1 to 200 MHz of the Synergy DQK-10-100S coupler showing the nominal 3 dB coupling loss (s-parameter S21, red trace) between ports 1 and 4, return loss (−S11, blue) of port 1, isolation (S21, green) of port 2 from port and phase difference between ports 3 and 4 (°, violet). The phase difference plot is the ratio [S21(port1 to port 4)/S21(port 1 to port 3)]. The coupling loss between ports 1 and 3 is the same as ports 1 and 4; ideally, the loss is 3.0 dB but the coupler is a practical device with slightly higher loss as seen in the marker table upper-left. The left vertical scale shows the units/division for each trace and the right vertical scale shows the reference position for each trace. The marker table shows values for each parameter at selected frequencies. All parameters are well within the manufacturer's specifications. Measurements were made with a DG8SAQ Vector Network Analyzer. (Image © 2014 W. Reeve)

## 3. Radio Wave Decomposition

As described in [ReevePol], a circularly polarized radio wave can be decomposed (or resolved) into two linearly polarized component waves that are perpendicular to each other (similarly, a linearly polarized radio wave can be decomposed into two oppositely rotating circularly polarized radio waves). In our application, two linearly polarized perpendicular antennas – crossed-dipoles – can be used to receive circular polarizations by combining the antenna outputs after phase shifting one of them by 90°. When a quadrature coupler is used to combine the outputs, the antenna system can discriminate the two rotation directions of the circularly polarized radio waves – one output provides a response only for RHCP and the other output provides a response only for LHCP. The two dipoles in the LWA crossed-dipole antenna are oriented north-south and east-west and, thus, are perpendicular. Each dipole is sensitive to the linear component of the incident radio wave that is aligned with it.

## 4. Signal Analysis

The analysis described in this section is simplified and assumes ideal conditions and devices. A much more detailed and complete analysis of quadrature couplers (and directional couplers, in general) may be found in [Matthaei]. For our purposes

- The plane of the incoming radio wave is perpendicular to the antenna axis (figure 3); for the LWA antenna the axis is along the center mast
- All received radio waves have the same amplitude
- The electrical waveform outputs from the two dipoles have the same amplitude
- All ports are terminated in their design impedance (perfectly matched)

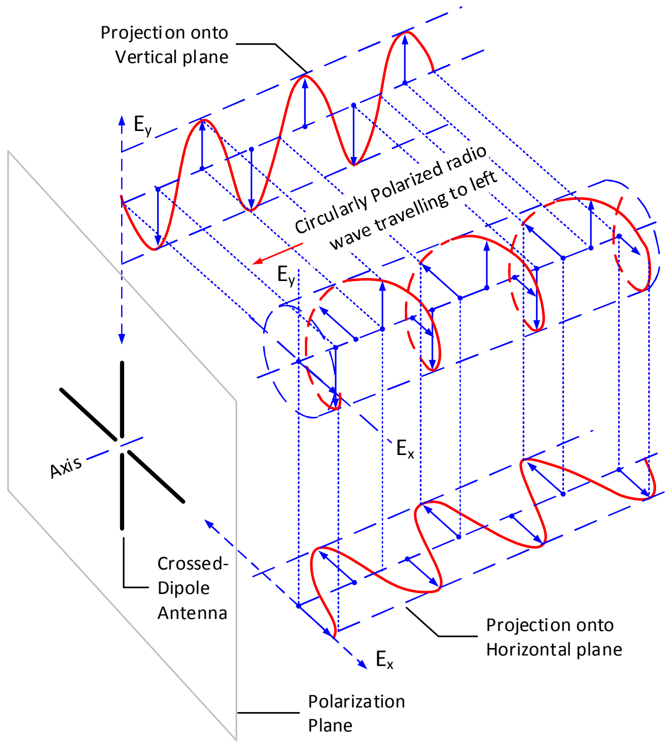


Figure 3 ~ Circularly polarized radio wave propagating from right-to-left toward a crossed-dipole antenna. The polarization plane is parallel to the plane of the crossed dipoles, shown here as vertical and horizontal wires, and perpendicular to the antenna axis. The circularly polarized radio wave has two linear components indicated by the projections onto horizontal and vertical planes. It should be noted that horizontal and vertical are arbitrary orientations. Other orientations can be used if they are perpendicular to each other. (Image © 2014 W. Reeve)

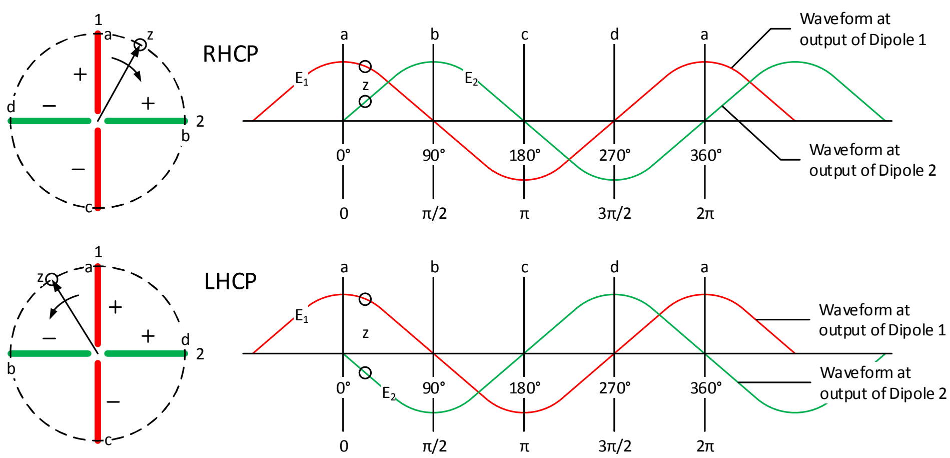


Figure 4 ~ Phase relationships of right-hand and left-hand circularly polarized radio waves. The radio source is above the plane of the page and transmitting toward it. The rotation of the radio wave electric field vector (phasor) traces the paths shown by the dashed lines superposed over the crossed-dipoles 1 and 2 shown on the left. The notations a, b, c and d are placed at 90° intervals of the phasor rotation for comparison. The notation z and associated circle show an arbitrary point. The corresponding waveforms at the output of each dipole are shown on the right. It is seen that by advancing or retarding one of the antenna outputs by 90° (π/2 rad), the two outputs reinforce or cancel each other depending on the rotation direction. (Image © 2014 W. Reeve)

With RHCP (figure 4), the electric field vector, phasor, of the radio wave will first excite the vertical dipole such that its output to port 1 of the coupler is $E_1 \cdot \cos(\omega \cdot t)$, where $E_1$ is the radio wave amplitude, $\omega$ is radian frequency ($\omega = 2 \cdot \pi \cdot f$ rad/s), *f* is frequency (Hz) and *t* is time (s). The phasor continues to rotate and it will excite the horizontal dipole when its phase is 90° ($\pi / 2$ rad) with respect to the vertical dipole. The waveform output from the horizontal dipole to coupler port 2 is $E_2 \cdot \cos(\omega \cdot t + \pi / 2)$. Therefore, with respect to the simplified block diagram (figure 5),

Port 1 incident voltage: $E_1 \cdot \cos(\omega \cdot t)$ (1)

Port 2 incident voltage: $E_2 \cdot \cos(\omega \cdot t + \pi / 2)$ (2)

The electrical waveforms from ports 1 and 2 are reduced by the factor $1/2$ (3 dB) in power or $1/\sqrt{2}$ in voltage as they are coupled to port 3. Furthermore, the coupler shifts the waveform from port 1 by 90° and from port 2 by 0°. Therefore,

Port 3 90° output voltage: $\frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi / 2) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi / 2 + 0)$ (3)

Using the trigonometric identity $\cos(\omega \cdot t + \pi / 2) = -\sin(\omega \cdot t)$ and noting that $E_1 = E_2 = E$, equation (3) may be simplified

Port 3 90° output voltage: $$\frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi / 2) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi / 2) = -\frac{E}{\sqrt{2}} \cdot \left[\sin(\omega \cdot t) + \sin(\omega \cdot t)\right]$$
$$= -\sqrt{2} \cdot E \cdot \sin(\omega \cdot t)$$ (4)

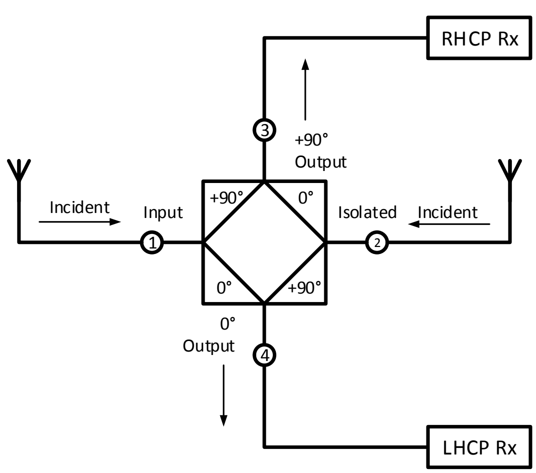


Figure 5 ~ Simplified block diagram of the quadrature coupler application showing port nomenclature. The designations for the two receivers, RHCP Rx and LHCP Rx, are based on the signal analysis. (Image © 2014 W. Reeve)

It is seen that the waveforms from the two dipoles for the RHCP radio wave reinforce (add) at the port 3 output, and the receiver connected to port 3 will indicate a response. Now, we will examine the electrical waveforms coupled to port 4. These are reduced by the same factors as before; however, the delays introduced by the coupler are reversed such that port 1 is delayed by 0° and port 2 is delayed by 90°, and

<u>Port 4 0° output voltage</u>: $\frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi/2 + \pi/2) = \frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi)$ (5)

In this case, we use the trigonometric identity $\cos(\omega \cdot t + \pi) = -\cos(\omega \cdot t)$, and

<u>Port 4 0° output voltage</u>: $\frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi) = \frac{E}{\sqrt{2}} \cdot \left[\cos(\omega \cdot t) - \cos(\omega \cdot t)\right]$
$= 0$ (6)

The waveforms from the two dipoles for the RHCP radio wave cancel each other at the port 4 output, and the corresponding receiver does not indicates a response. Our next task is to examine the coupler inputs and outputs for a radio wave with LHCP. We will continue to use port 1 as the phase reference. For LHCP, port 2 input will be advanced in phase by 90° compared to port 1, and

<u>Port 1 input voltage</u>: $E_1 \cdot \cos(\omega \cdot t)$ (7)

<u>Port 2 input voltage</u>: $E_2 \cdot \cos(\omega \cdot t - \pi/2)$ (8)

We again analyze the waveforms as they are coupled to output ports 3 and 4. At port 3 output, the waveform from port 1 is delayed by 90° and from port 2 is delayed by 0°, and we have

<u>Port 3 90° Output voltage</u>: $\frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t + \pi/2) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t - \pi/2)$ (9)

Using the trigonometric identities

$$\cos(\omega \cdot t + \pi/2) = -\sin(\omega \cdot t)$$
$$\cos(\omega \cdot t - \pi/2) = +\sin(\omega \cdot t)$$

<u>Port 3 90° output voltage</u>: $\frac{E}{\sqrt{2}} \cdot \left[-\sin(\omega \cdot t) + \sin(\omega \cdot t)\right]$
$= 0$ (10)

The phases of the waveforms from the two input ports 1 and 2 coupled to port 4 output are 0° and 90°, respectively, and

<u>Port 4 0° Output voltage</u>: $\frac{E_1}{\sqrt{2}} \cdot \cos(\omega \cdot t) + \frac{E_2}{\sqrt{2}} \cdot \cos(\omega \cdot t - \pi/2 + \pi/2) = \frac{E}{\sqrt{2}} \cdot \left[\cos(\omega \cdot t) + \cos(\omega \cdot t)\right]$
$= \sqrt{2} \cdot E \cdot \cos(\omega \cdot t)$ (11)

These analyses show that port 3 output is cancelled and port 4 output is reinforced for LHCP and port 3 output is reinforced and port 4 output is cancelled for RHCP (table 2). Thus, we have shown that the coupler discriminates between the two rotation directions, and we can label coupler port 3 as *RHCP* and port 4 as *LHCP* for the connections as they were defined above.

Table 2 ~ Signal analysis summary

| Polarization | 1: IN | 2: ISO | 3: +90° OUT | 4: 0° OUT |
|---|---|---|---|---|
| **RHCP** | $E\cdot\cos(\omega\cdot t)$ | $E\cdot\cos(\omega\cdot t+\pi/2)$ | $-\sqrt{2}\cdot E\cdot\sin(\omega\cdot t)$ | 0 |
| **LHCP** | $E\cdot\cos(\omega\cdot t)$ | $E\cdot\cos(\omega\cdot t-\pi/2)$ | 0 | $\sqrt{2}\cdot E\cdot\cos(\omega\cdot t)$ |

## 5. Impedance Mismatch Analysis

We are concerned with impedance mismatches because they cause reflections. Reflected energy represents energy lost to detection. Also, if the reflections are coupled back to the antennas, they may radiate and cause radio frequency interference. In this section we consider only impedance mismatches at the output ports, in particular mismatches caused by the instruments connected to the antennas through the quadrature coupler. <u>It should be noted that radiation by the LWA crossed-dipole antenna of these reflections is not possible because it has an active balun that prevents signals on its output from appearing at the antenna itself (that is, there is no coupling by the active balun from its output to input); however, radiation is possible from ordinary passive crossed-dipole antennas.</u>

We investigate the effects of mismatch by starting with the same input signals as in the previous section, and we assume the inputs are perfectly matched but the outputs are mismatched. The waveforms from input ports 1 and 2 are split as previously described but a portion is reflected back by the mismatches at output ports 3 and 4. The amount of reflection from each output port is determined by its *reflection coefficient* $\Gamma$. The reflection coefficient is defined as the ratio of the reflected voltage to the incident voltage but it also may be equivalently defined in terms of impedances, as in

$$\Gamma=\frac{E_R}{E_I}=\frac{Z_L-Z_0}{Z_L+Z_0} \tag{12}$$

where

$E_R$ Reflected voltage
$E_I$ Incident voltage
$Z_L$ Load impedance
$Z_0$ Network or device characteristic impedance

For RHCP and the setup shown (figure 6),

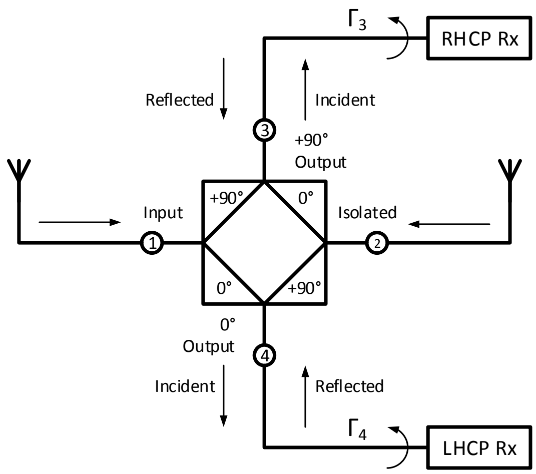


Figure 6 ~ Block diagram showing reflections at the coupler output ports 3 and 4. The waveforms from the two antennas are coupled and delayed as described in the previous section. A portion is reflected at each output port according to the reflection coefficients $\Gamma_3$ and $\Gamma_4$. (Image © 2014 W. Reeve)

Port 3 reflected voltage for RHCP:

$$\Gamma_3 \cdot \left[ -\sqrt{2} \cdot E \cdot \sin(\omega \cdot t) \right] \tag{13}$$

and

Port 4 reflected voltage for RHCP:

$$\Gamma_4 \cdot 0 = 0 \tag{14}$$

Similarly, for LHCP

Port 3 reflected voltage for LHCP:

$$\Gamma_3 \cdot 0 = 0 \tag{15}$$

and

Port 4 reflected voltage for LHCP:

$$\Gamma_4 \cdot \sqrt{2} \cdot E \cdot \cos(\omega \cdot t) \tag{16}$$

The reflected voltages are again reduced by the factor $1/\sqrt{2}$ and delayed as they are coupled back to the input ports 1 and 2. The reflection from port 3 is delayed by 90° when it is coupled back to port 1 and 0° back to port 2. Therefore, for RHCP

Port 1 output voltage from reflection at port 3 for RHCP:

$$\Gamma_3 \cdot \left[ -\frac{\sqrt{2}}{\sqrt{2}} \cdot E \cdot \sin(\omega \cdot t + \pi/2) \right] = -\Gamma_3 \cdot E \cdot \sin(\omega \cdot t + \pi/2) \tag{17}$$

Using the trigonometric identity $\sin(\omega \cdot t + \pi/2) = \cos(\omega \cdot t)$,

Port 1 output voltage from reflection at port 3 for RHCP:

$$-\Gamma_3 \cdot E \cdot \cos(\omega \cdot t) \tag{18}$$

For LHCP

Port 1 output voltage from reflection at port 3 for LHCP:

$$\Gamma_3 \cdot 0 = 0 \tag{19}$$

Similarly, the voltages at port 2 due to reflections at port 4 are

Port 2 output voltage from reflection at port 4 for RHCP:

$$\Gamma_4 \cdot 0 = 0$$

Port 2 output voltage from reflection at port 4 for LHCP:

$$\Gamma_4 \cdot \left[ \frac{\sqrt{2}}{\sqrt{2}} \cdot E \cdot \cos(\omega \cdot t + \pi/2) \right] = \Gamma_4 \cdot \left[ E \cdot \cos(\omega \cdot t + \pi/2) \right] \tag{20}$$

Using the trigonometric identity $\cos(\omega \cdot t + \pi/2) = -\sin(\omega \cdot t)$,

Port 2 0° output voltage from reflection at port 4 for LHCP:

$$-\Gamma_4 \cdot E \cdot \sin(\omega \cdot t) \tag{21}$$

The foregoing impedance mismatch analysis is summarized (table 3).

Table 3 ~ Mismatch analysis summary. Upper panel: For mismatch analysis, ports 3 and 4 act as inputs and ports 1 and 2 act as outputs for the reflected signals. Lower panel: Original input and output signals for comparison.

| **Polarization** | **1: IN (reflection)** | **2: ISO (reflection)** | **3: +90° Reflection** | **4: 0° Reflection** |
|---|---|---|---|---|
| **RHCP** | $-\Gamma_3 \cdot E \cdot \cos(\omega \cdot t)$ | 0 | $\Gamma_3 \cdot \left[ -\sqrt{2} \cdot E \cdot \sin(\omega \cdot t) \right]$ | 0 |
| **LHCP** | 0 | $-\Gamma_4 \cdot E \cdot \sin(\omega \cdot t)$ | 0 | $\Gamma_4 \cdot \sqrt{2} \cdot E \cdot \cos(\omega \cdot t)$ |

| **Polarization** | **1: IN (signal)** | **2: ISO (signal)** | **3: +90° OUT** | **4: 0° OUT** |
|---|---|---|---|---|
| **RHCP** | $E \cdot \cos(\omega \cdot t)$ | $E \cdot \cos(\omega \cdot t + \pi/2)$ | $-\sqrt{2} \cdot E \cdot \sin(\omega \cdot t)$ | 0 |
| **LHCP** | $E \cdot \cos(\omega \cdot t)$ | $E \cdot \cos(\omega \cdot t - \pi/2)$ | 0 | $\sqrt{2} \cdot E \cdot \cos(\omega \cdot t)$ |

The above mismatch analysis is based on the reflection coefficient. A related parameter is *return loss*, a logarithmic ratio that may be derived from the reflection coefficient, as in

$$RL(dB) = 20\log\left|\frac{1}{\Gamma}\right| = 20\log\left|\frac{E_I}{E_R}\right| \quad (13)$$

A smaller reflection coefficient indicates a better impedance match whereas a larger return loss indicates a better impedance match.

We are concerned with radiation by the antennas of the reflections coupled back from the output ports. As previously noted, the LWA antenna cannot radiate reflections coupled to it by the quadrature coupler. However, in the general case of an ordinary crossed-dipole or other completely passive antenna, the possibility of radiation exists. The following paragraphs discuss two types of equipment connected to an ordinary antenna, the Callisto instrument by itself and the Up-Converter model UPC-1 operating in the 20 to 90 MHz range.

On the basis of 50 ohms impedance, the measured return loss of the Callisto instrument varies with frequency (figure 7). It is typically 10 dB across the 10 to 100 MHz band of the antenna but decreases substantially at frequencies below Callisto's low frequency limit of 45 MHz. Similar measurements of the UPC-1 up-converter show a typical return loss of 15 dB (figure 8).

When used with an ordinary crossed-dipole antenna (not the LWA antenna), an attenuator can be used on the Callisto RF input or up-converter RF input to improve the return loss (at the expense of instrument sensitivity). For example, a 10 dB attenuator would improve the return loss by 20 dB (twice the attenuator value because both the incident and reflected waveforms are attenuated). Therefore, a 10 dB attenuator would improve the return loss to > 30 dB, corresponding to a reflection coefficient < 0.03. The return loss of the attenuator itself should be > 30 dB at the frequencies of interest so that it does not degrade the impedance matching.

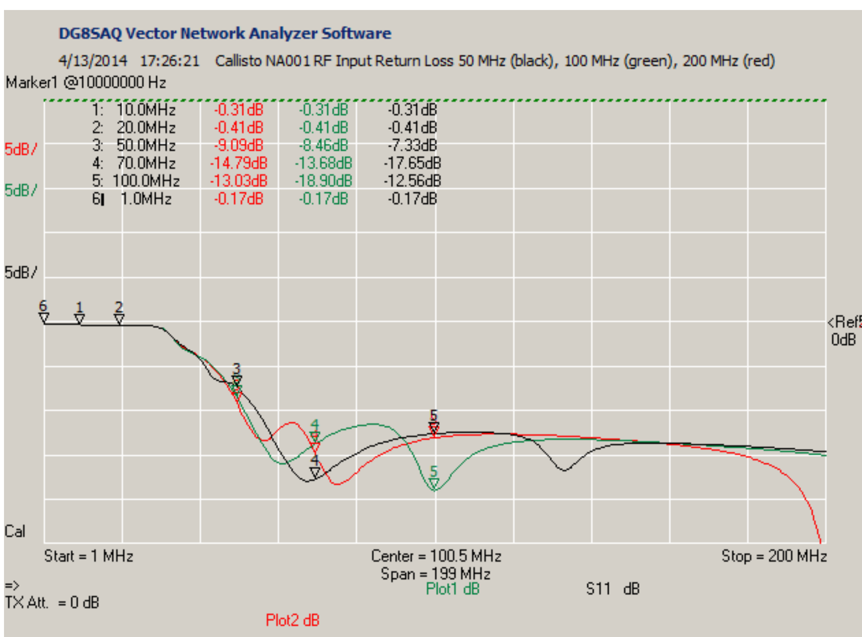


Figure 7 ~ Measured Callisto RF input return loss (−s11 parameter) while tuned to 50 MHz (black), 100 MHz (green) and 200 MHz (red). The marker table at upper-left indicates measured data at various frequencies. In the frequency band 10 to 100 MHz, the return loss varies from approximately 0 dB to 18 dB but typically *RL* > 10 dB corresponding to a reflection coefficient < 0.316.

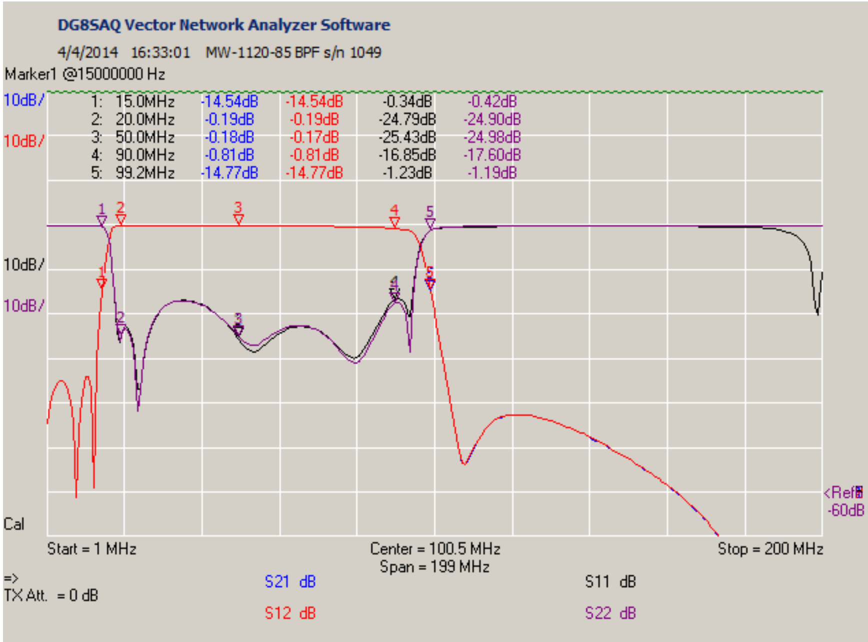


Figure 8 ~ Measured UPC-1 up-converter input return loss (−s11 parameter shown by black trace). This measurement is of the bandpass filter at the input to the up-converter. In the frequency band 10 to 100 MHz, the return loss varies from approximately 0 dB to 30 dB but typically $RL > 15$ dB corresponding to a reflection coefficient < 0.178. Other parameters are forward transmission loss (s21, blue behind red trace), backward transmission loss (s12, red) and output return loss (−s22, violet).

## 6. Long Wavelength Array Antenna Application

To ensure commonality in Long Wavelength Array antenna installations, the identification and nomenclature described in this section is the recommended standard for antennas purchased from {RvLWA} (table 4 and figure 9). It is possible that after operational experience is gained and results are compared to observatories that monitor solar radio emissions, the RHCP and LCHP labeling of the coupler output may need to be reversed. This will be handled by a revision to this document.

Table 4 ~ Quadrature coupler nomenclature for Synergy DQK-10-100S. Applies to Rev. 1.x of this document.

| Coupler port number | Coupler port name | Connect to | Remarks |
|---|---|---|---|
| **1** | Input | North-South Antenna | Active balun SMA connector marked **N-S** |
| **2** | Isolated | East-West Antenna | |
| **3** | +90° Output | RHCP Receiver | |
| **4** | 0° Output | LHCP Receiver | |

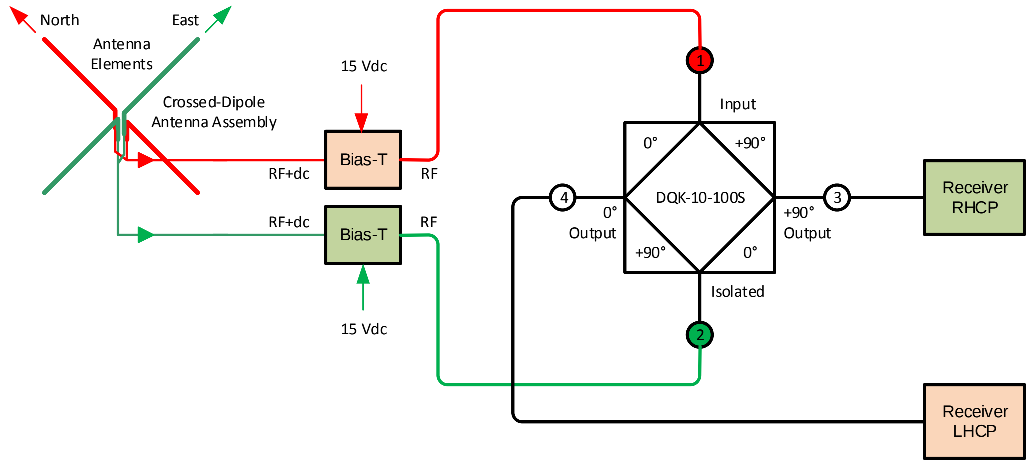


Figure 9 ~ LWA antenna system connection diagram. (Image © 2014 W. Reeve)

## 7. Conclusions

A quadrature coupler application is described that enables reception of circularly polarized radio waves by a crossed-dipole antenna such as the LWA antenna and two receivers. The coupler discriminates right-hand and left-hand circular polarization and provides a separate output for each. One risk of using the coupler in this application is radiation of reflections from the output ports due to mismatch, but this can be reduced by ensuring high return loss (low reflection coefficient) at the output connections.

## 8. References and Links

[Matthaei] Matthaei, G., Young, L., Jones, E., Microwave Filters, Impedance-Matching Networks, and Coupling Structures, Artech House, Inc., 1980

[ReeveLWA] Reeve, W., Modeling the Long Wavelength Array Crossed-Dipole Antenna, *Radio Astronomy*, Society of Amateur Radio Astronomers, January-February 2014

[ReevePol] Reeve, W., Introduction to Radio Wave Polarization, *Radio Astronomy*, Society of Amateur Radio Astronomers, May-June 2014

{e-Callisto} http://www.e-callisto.org/

{RvLWA} http://www.reeve.com/RadioScience/Antennas/ActiveCrossed-Dipole/ActiveBalunOrderInfo.htm

**Document information**

Author: Whitham D. Reeve

Revision: 0.0 (Original draft started, 08 Apr 2014)
0.1 (1st draft completed, 12 Apr 2014)
0.2 (Corrected mismatch and return loss, 14 Apr 2014)
0.3 (2nd draft completed, 17 Apr 2014)
1.0 (Distribution with minor updates, 25 Apr 2014)
1.1 (Corrected DQK-10-100S part number, 26 Apr 2014)
1.2 (Minor revisions, 3 May 2014)
1.3 (Completed for distribution, 31 May 2014)
1.4 (Corrected table 4, 28 Jul 2014)